

Green's Functions in Perturbative Quantum Gravity

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We show that the Green's functions in non-linear gauge in the theory of perturbative quantum gravity is expressed as a series in terms of those in linear gauges. This formulation also holds for operator Green's functions. We further derive the explicit relation between the Green's functions in the theory of perturbative quantum gravity in a pair of arbitrary gauges. This process involves some sort of modified FFBRST transformations which is derivable from infinitesimal field-dependent BRST transformations.

I. INTRODUCTION

Since its inception, the general relativity has many striking similarities to gauge theories. For instance, both involve the idea of local symmetry and therefore share a number of formal properties. Moreover, consistent quantum gauge theories are well-established but as yet no satisfactory quantum field theory of gravity has been investigated. The structures of the Lagrangians of these theories are rather different. The Yang-Mills Lagrangian contains only up to four-point interactions while the Einstein-Hilbert Lagrangian contains infinitely many interactions. Despite these differences, string theory provides us sufficient reasons, on the basis of which it can be claimed that gravity and gauge theories can, in fact, be unified. For example, the Maldacena conjecture [1, 2] relates the weak coupling limit of a gravity theory to a strong coupling limit of a special supersymmetric gauge field theory. With this similarity, the gauge theories are allowed to be used directly as a resource for computations in perturbative quantum gravity.

The perturbative quantum gravity as a gauge theory is a subject of extensive research interests [3–5]. For examples, the mode analysis and Ward identities for a ghost propagator for perturbative quantum gravity has been demonstrated [6]. The Feynman rules and propagator for gravity in the physically interesting cases of inflation have been analysed [7]. The propagator for a gauge theory exists only after fixing a gauge. For instance, the Landau and Curci-Ferrari type gauges have their common uses in the perturbation theory [8, 9]. Being gauge-fixed, the theory loses their local gauge invariance. However, it possesses the rather different fermionic rigid BRST invariance [10, 11].

The BRST symmetry and the associated concept of BRST cohomology provide the most used covariant quantization method for constrained systems such as gauge and string theories [12, 13]. The BRST and the anti-BRST symmetries for perturbative quantum gravity in flat spacetime have also been investigated [14–16] which was summarized by N. Nakanishi and I. Ojima [17]. Recently, the BRST formulation for the perturbative quantum gravity in general curved spacetime has also been analyzed [18, 19]. The usual infinitesimal BRST transformation has been generalized by allowing the parameter finite and field-dependent [20]. This FFBRST enjoys the properties of usual BRST except it does not leave the path integral measure invariant. The FFBRST transformations have found several applications in gauge field theories in flat spacetime [20–32] as well as in curved spacetime [33]. The FFBRST formulation to connect the Green's function of Yang-Mills theory in a set of two otherwise unrelated gauge choices has been established [35]. Nevertheless, the FFBRST formulation to connect Green's functions has not been developed so far in the context of perturbative quantum gravity. The development of FFBRST formulation to connect Green's functions in perturbative quantum gravity is goal of present investigation.

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In this paper, we discuss the usual FFBRST transformation in perturbative quantum gravity to connect the linear and non-linear gauges of the theory. Further, we establish a connection between arbitrary Green's functions (or operator Green's functions) in two sets of gauges for the theory of perturbative quantum gravity. In view of their extreme importance, we choose these to be the linear (Landau) and non-linear (Curci-Ferrari) type gauges. Here we find that to connect the Green's functions of the theory rather than connection of gauges we require different FFBRST transformation. Finally, we establish a compact result expressing an arbitrary Green's function or operator Green's function in non-linear gauges with a closed expression involving similar Green's functions in Landau gauges.

This paper is presented as follows. In Sec. II, we present the usual FFBRST transformation for a general gauge theory. In Sec. III, we recapitulate the FFBRST transformation to connect the linear and non-linear gauges in linearized gravity. In Sec. IV, we demonstrate the similar FFBRST transformation to connect the Green's functions of the perturbative quantum gravity by a compact formula. In the last section, we summarize the results with future motivations.

II. THE USUAL FFBRST TRANSFORMATIONS

In this subsection, we recapitulate the FFBRST transformation for the general gauge theory in general curved spacetime [36]. For this purpose, we first write the usual BRST transformation

$$\delta_b \phi(x) = s\phi(x)\delta\Lambda, \quad (1)$$

where $\delta\Lambda$ is infinitesimal and field-independent Grassmann parameter and $\phi(x)$ is the generic notation of fields (h, c, \bar{c}, b) involved the theory of quantum gravity. The observations of BRST transformation that its basic properties do not depend on whether the parameter $\delta\Lambda$ is (i) finite or infinitesimal, (ii) field-dependent or not, as long as it is anticommuting and spacetime independent. This renders us a freedom to make the parameter, $\delta\Lambda$ finite and field-dependent without affecting its basic features. The first step towards the goal is to make the infinitesimal parameter field-dependent by interpolating a continuous parameter, κ ($0 \leq \kappa \leq 1$), in the theory. The fields, $\phi(x, \kappa)$, depend on κ such that $\phi(x, \kappa = 0) = \phi(x)$ is the initial fields and $\phi(x, \kappa = 1) = \phi'(x)$ is the transformed fields.

The infinitesimal field-dependent BRST transformation is defined by [20]

$$d\phi(x, \kappa) = s[\phi(x)]\Theta'[\phi(\kappa)]d\kappa, \quad (2)$$

where the $\Theta'[\phi(\kappa)]d\kappa$ is the infinitesimal but field-dependent parameter. The FFBRST transformation is then prevailed by integrating this infinitesimal transformation from $\kappa = 0$ to $\kappa = 1$, as follows

$$\phi' \equiv \phi(x, \kappa = 1) = \phi(x, \kappa = 0) + s[\phi(x)]\Theta[\phi], \quad (3)$$

where

$$\Theta[\phi] = \Theta'[\phi] \frac{\exp f[\phi] - 1}{f[\phi]}, \quad (4)$$

is the finite field-dependent parameter and $f[\phi]$ is given by

$$f[\phi] = \sum_i \int d^4x \frac{\delta\Theta'[\phi]}{\delta\phi_i(x)} s_b \phi_i(x). \quad (5)$$

The resulting FFBRST transformation leaves the effective action invariant but the functional integral changes non-trivially under it [20]. Now we compute the Jacobian of path integral measure under the FFBRST transformation.

We first define the Jacobian of the path integral measure under such transformations with an arbitrary finite field-dependent parameter, $\Theta[\phi(x)]$, as

$$\mathcal{D}\phi' = J(\kappa)\mathcal{D}\phi(\kappa). \quad (6)$$

The Jacobian, $J(\kappa)$, can be replaced within the functional integral as

$$J(\kappa) \rightarrow \exp[iS_1[\phi(x, \kappa), \kappa]], \quad (7)$$

where $S_1[\phi(x), \kappa]$ is local functional of fields, iff the following condition gets satisfied [20]

$$\int \mathcal{D}\phi \left[\frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1[\phi(x, \kappa), \kappa]}{d\kappa} \right] e^{i(S_L[\phi] + S_1[\phi, \kappa])} = 0. \quad (8)$$

The infinitesimal change in the Jacobian $J(\kappa)$ is addressed with the following formula [20]

$$\frac{1}{J} \frac{dJ}{d\kappa} = - \int d^4y \left[\pm s\phi(y, \kappa) \frac{\delta\Theta'[\phi]}{\delta\phi(y, \kappa)} \right], \quad (9)$$

where sign $+$ is used for bosonic fields ϕ and $-$ sign is used for fermionic fields ϕ .

Recently, exactly similar FFBRST transformations have also been considered and general Jacobian is calculated explicitly in terms of the general finite parameter Θ [37].

III. THE FFBRST TRANSFORMATION IN PERTURBATIVE QUANTUM GRAVITY: PRELIMINARIES

In this section we consider perturbative quantum gravity in the framework of FFBRST transformation. In particular we analyse the perturbative quantum gravity in linear and non-linear gauges. Then we generalize the BRST transformation by making the transformation finite and field-dependent. Furthermore, we establish the connection between these two gauges using FFBRST transformation [36].

A. The linearized quantum gravity

Let us start by writing the classical Lagrangian density for gravity in general curved spacetime

$$\mathcal{L}_c = \sqrt{-g}(R - 2\lambda), \quad (10)$$

where R is Ricci scalar curvature and λ is a cosmological constant. Here units are setted in such a manner that $16\pi G = 1$. In the weak approximation the full metric g_{ab}^f can be written as a sum of fixed metric of background spacetime g_{ab} and the small perturbations around it, denoted by h_{ab} . This fluctuation is considered as a quantum field that needs to be quantized. Therefore, numerically

$$g_{ab}^f = g_{ab} + h_{ab}. \quad (11)$$

Incorporating such decomposition, the Lagrangian density given in (10) described in terms of h_{ab} remains invariant under the following coordinate transformation:

$$\delta_\Lambda h_{ab} = \nabla_a \Lambda_b + \nabla_b \Lambda_a + \mathcal{L}_{(\Lambda)} h_{ab}, \quad (12)$$

where the Lie derivative of h_{ab} with respect to the vector field Λ_a is defined by

$$\mathcal{L}_{(\Lambda)} h_{ab} = \Lambda^c \nabla_c h_{ab} + h_{ac} \nabla_b \Lambda^c + h_{cb} \nabla_a \Lambda^c. \quad (13)$$

The gauge invariance reflects the redundancy in physical degrees of freedom. Such redundancy in gauge degrees of freedom produces constraints in the canonical quantization and leads divergences in the generating functional. In order to fix the redundancy we choose the following gauge-fixing condition satisfied by quantum field:

$$G[h]_a = (\nabla^b h_{ab} - \beta \nabla_a h) = 0, \quad (14)$$

where the parameter $\beta \neq 1$. Because $\beta = 1$ leads to vanishing conjugate momentum corresponding to h_{00} and therefore generating functional diverges. This gauge-fixing condition quantum level by adding following term in the classical action:

$$\mathcal{L}_{gf} = \sqrt{-g}[ib^a(\nabla^b h_{ab} - \beta \nabla_a h)]. \quad (15)$$

The induced (Faddeev–Popov) ghost term is then defined by

$$\mathcal{L}_{gh} = \sqrt{-g}\bar{c}^a M_{ab} c^b, \quad (16)$$

where Faddeev–Popov matrix operator M_{ab} has following expression:

$$M_{ab} = i\nabla_c[\delta_b^c \nabla_a + g_{ab} \nabla^c - 2\beta \delta_a^c \nabla_b + \nabla_b h_a^c - h_{ab} \nabla^c - h_b^c \nabla_a - \beta g_a^c g^{ef}(\nabla_b h_{ef} + h_{eb} \nabla_f + h_{fb} \nabla_e)]. \quad (17)$$

Henceforth, the effective action for perturbative quantum gravity in curved spacetime dimensions (in linear gauge) reads

$$S_L = \int d^4x (\mathcal{L}_c + \mathcal{L}_{gf} + \mathcal{L}_{gh}), \quad (18)$$

which is invariant under following BRST transformations:

$$s h_{ab} = (\nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}), \quad s c^a = -c_b \nabla^b c^a, \quad s \bar{c}^a = b^a, \quad s b^a = 0. \quad (19)$$

Here we observe that the gauge-fixing and the ghost parts of the effective Lagrangian density are BRST-exact. Therefore,

$$\begin{aligned} \mathcal{L}_g &= \mathcal{L}_{gf} + \mathcal{L}_{gh}, \\ &= i s \sqrt{-g} [\bar{c}^a (\nabla^b h_{ab} - \beta \nabla_a h)], \\ &= s \Psi. \end{aligned} \quad (20)$$

The gauge-fixed fermion (Ψ) then has the expression

$$\Psi = i \sqrt{-g} [\bar{c}^a (\nabla^b h_{ab} - \beta \nabla_a h)]. \quad (21)$$

However, the gauge-fixing and ghost terms in non-linear Curci–Ferrari gauge condition are written by

$$\begin{aligned} \mathcal{L}'_g &= \mathcal{L}'_{gf} + \mathcal{L}'_{gh}, \\ &= \sqrt{-g} \left[ib^a (\nabla^b h_{ab} - \beta \nabla_a h) - i \bar{c}^b \nabla_b c^a (\nabla^b h_{ab} - \beta \nabla_a h) + \bar{c}^a M_{ab} c^b + \frac{\alpha}{2} b^b \nabla_b \bar{c}^a c_a \right. \\ &\quad - \frac{\alpha}{2} \bar{c}^c \nabla_c b^b \nabla_b \bar{c}^a c_a - \frac{\alpha}{2} \bar{b}^b \nabla_b b^a c_a - \frac{\alpha}{2} \bar{c}^b \nabla_b \bar{c}^a c_d \nabla^d c_a - \frac{\alpha}{2} b_a b^a + \alpha \bar{c}^b b^b \nabla_b c_a \\ &\quad \left. + \alpha \bar{c}^a \bar{c}^b c^d \nabla_b \nabla_d c_a \right], \end{aligned} \quad (22)$$

where α is a gauge parameter. For instance, the effective action, having such gauge-fixing and Faddeev–Popov ghost terms, in non-linear gauge is given by

$$S_{NL} = \int d^4x (\mathcal{L}_c + \mathcal{L}'_g), \quad (23)$$

which remains unchanged under following BRST transformations:

$$\begin{aligned} s h_{ab} &= \nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}, \\ s c^a &= -c_b \nabla^b c^a, \\ s \bar{c}^a &= b^a - \bar{c}^b \nabla_b c^a, \\ s b^a &= -b^b \nabla_b c^a - \bar{c}^b c^d \nabla_b \nabla_d c^a. \end{aligned} \quad (24)$$

B. FFBRST transformation for linear to non-linear gauge

We construct the FFBRST transformation for perturbative quantum gravity utilizing the BRST transformation (19) as follows

$$\begin{aligned} f h_{ab} &= (\nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}) \Theta[\phi], \\ f c^a &= -c_b \nabla^b c^a \Theta[\phi], \\ f \bar{c}^a &= b^a \Theta[\phi], \\ f b^a &= 0, \end{aligned} \quad (25)$$

where $\Theta[\phi]$ is an arbitrary finite field-dependent parameter. To establish the connection between the Landau and the (non-linear) Curci–Ferrari gauge we opt the finite field-dependent parameter constructed from following infinitesimal field-dependent parameter:

$$\Theta'[\phi] = i \frac{\alpha}{2} \sqrt{-g} \int d^4 y (\bar{c}_b \nabla^b \bar{c}^a c_a - \bar{c}^a b_a - \bar{c}^a \bar{c}_b \nabla^b c_a). \quad (26)$$

Exploiting relations (9) and (26) we calculate the change in Jacobian as

$$\begin{aligned} \frac{1}{J(\kappa)} \frac{dJ(\kappa)}{d\kappa} &= -i \frac{\alpha}{2} \sqrt{-g} \int d^4 x [-b_b \nabla^b \bar{c}^a c_a + \bar{c}^d \nabla_d c_b \nabla^b \bar{c}^a c_a + \bar{c}_b \nabla^b b^a c_a + \bar{c}_b \nabla^b \bar{c}^a c_d \nabla^d c_a \\ &\quad + b_a b^a - 2\bar{c}^a b_b \nabla^b c_a - 2\bar{c}^a \bar{c}_b c_d \nabla^b \nabla^d c_a]. \end{aligned} \quad (27)$$

The local functional S_1 in the expression (7) is written by

$$\begin{aligned} S_1[\phi(\kappa), \kappa] &= \int d^4 x [\xi_1 b_b \nabla^b \bar{c}^a c_a + \xi_2 \bar{c}^d \nabla_d c_b \nabla^b \bar{c}^a c_a + \xi_3 \bar{c}_b \nabla^b b^a c_a + \xi_4 \bar{c}_b \nabla^b \bar{c}^a c_d \nabla^d c_a \\ &\quad + \xi_5 b_a b^a + \xi_6 \bar{c}^a b_b \nabla^b c_a + \xi_7 \bar{c}^a \bar{c}_b c_d \nabla^b \nabla^d c_a], \end{aligned} \quad (28)$$

where parameters ξ_i ($i = 1, 2, \dots, 7$) depend explicitly on parameter κ as follows [36],

$$\begin{aligned} \xi_1 &= \frac{\alpha}{2} \sqrt{-g} \kappa, \quad \xi_2 = -\frac{\alpha}{2} \sqrt{-g} \kappa, \quad \xi_3 = -\frac{\alpha}{2} \sqrt{-g} \kappa, \quad \xi_4 = -\frac{\alpha}{2} \sqrt{-g} \kappa, \\ \xi_5 &= -\frac{\alpha}{2} \sqrt{-g} \kappa, \quad \xi_6 = \alpha \sqrt{-g} \kappa, \quad \xi_7 = \alpha \sqrt{-g} \kappa. \end{aligned} \quad (29)$$

With these identifications of $\xi_i(\kappa)$ the expression of S_1 becomes

$$\begin{aligned} S_1[\phi(\kappa), \kappa] &= \kappa \int d^4 x \sqrt{-g} \left[\frac{\alpha}{2} b_b \nabla^b \bar{c}^a c_a - \frac{\alpha}{2} \bar{c}^d \nabla_d c_b \nabla^b \bar{c}^a c_a - \frac{\alpha}{2} \bar{c}_b \nabla^b b^a c_a - \frac{\alpha}{2} \bar{c}_b \nabla^b \bar{c}^a c_d \nabla^d c_a \right. \\ &\quad \left. - \frac{\alpha}{2} b_a b^a + \alpha \bar{c}^a b_b \nabla^b c_a + \alpha \bar{c}^a \bar{c}_b c_d \nabla^b \nabla^d c_a \right]. \end{aligned} \quad (30)$$

Therefore, the FFBRST transformation (25) changes the effective action within functional integration as

$$\begin{aligned} S_L + S_1(\kappa = 1) &= \int d^4 x [\mathcal{L}_c + i \sqrt{-g} b^a (\nabla^b h_{ab} - \beta \nabla_a h) + \sqrt{-g} \bar{c}^a M_{ab} c^b \\ &\quad + \frac{\alpha}{2} \sqrt{-g} b_b \nabla^b \bar{c}^a c_a - \frac{\alpha}{2} \sqrt{-g} \bar{c}^d \nabla_d c_b \nabla^b \bar{c}^a c_a - \frac{\alpha}{2} \sqrt{-g} \bar{c}_b \nabla^b b^a c_a - \frac{\alpha}{2} \sqrt{-g} \bar{c}_b \nabla^b \bar{c}^a c_d \nabla^d c_a \\ &\quad - \frac{\alpha}{2} \sqrt{-g} b_a b^a + \alpha \sqrt{-g} \bar{c}^a b_b \nabla^b c_a + \alpha \sqrt{-g} \bar{c}^a \bar{c}_b c_d \nabla^b \nabla^d c_a]. \end{aligned} \quad (31)$$

After performing a shift in the Nakanishi–Lautrup field by $\bar{c}^b \nabla_b c^a$, the above expression reduces to

$$\begin{aligned} S_L + S_1(\kappa = 1) &= \int d^4 x [\mathcal{L}_c + i \sqrt{-g} b^a (\nabla^b h_{ab} - \beta \nabla_a h) - i \sqrt{-g} \bar{c}^b \nabla_b c^a (\nabla^b h_{ab} - \beta \nabla_a h) \\ &\quad + \sqrt{-g} \bar{c}^a M_{ab} c^b], \\ &= S_{NL}, \end{aligned} \quad (32)$$

which is nothing but the effective action for perturbative quantum gravity in Landau gauge.

IV. RELATION BETWEEN GREEN'S FUNCTION FOR LINEAR AND NON-LINEAR GAUGES

In this section, we establish a procedure for FFBRST transformation that transforms the generating functional (Green's function) in one kind of a gauge choice to the generating functional in another kind of a gauge choice. For this purpose we define the generating functional for perturbative quantum gravity in linear gauge

$$W_L = \int \mathcal{D}\phi e^{iS_L[\phi]}, \quad (33)$$

which transforms under FFBRST transformation $\phi'(x) = \phi(x) + s\phi\Theta[\phi]$ defined in (25) as follows:

$$W_{NL} = \int \mathcal{D}\phi' e^{iS_L[\phi']} = W_L. \quad (34)$$

Now, we want to implement this transformation to connect the Greens functions in the two gauges for quantum gravity theory. According to the standard procedure, n -point Green's functions in non-linear gauge under FFBRST transformation transform as

$$\begin{aligned} G_{i_1 \dots i_n}^{NL} &= \int \mathcal{D}\phi' \prod_{r=1}^n \phi'_{i_r} e^{iS_{NL}[\phi']}, \\ &= \int \mathcal{D}\phi \prod_{r=1}^n (\phi_{i_r} + s_{i_r} \phi \Theta[\phi]) e^{iS_L[\phi]}, \\ &= G_{i_1 \dots i_n}^L + \Delta G_{i_1 \dots i_n}^L, \end{aligned} \quad (35)$$

where $\Delta G_{i_1 \dots i_n}^L$, refers the difference between the n -point Green's functions in the two sets of gauges. This may involve additional vertices corresponding to insertions of operators $s_{i_r} \phi$. But it seems technically incorrect for the following reasons.

A priori, it is not obvious that if condition (8) (for replacing Jacobian to e^{iS_1}) holds for quantum gravity then an equation modified to include an arbitrary operator $\mathcal{O}[\phi]$ of type

$$\int \mathcal{D}\phi \mathcal{O}[\phi] \left[\frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1[\phi(x, \kappa), \kappa]}{d\kappa} \right] e^{i(S_L[\phi] + S_1[\phi, \kappa])} = 0, \quad (36)$$

would also hold. Of course it does not hold in general for the reason discussed in [35]. For this reason, to connect the Green's functions for the two type of gauges we need a elaborate treatment of FFBRST transformation.

We begin with a general Green's function in non-linear gauge defined by

$$G = \int \mathcal{D}\phi' \mathcal{O}[\phi'] e^{iS_{NL}[\phi']}, \quad (37)$$

where $\mathcal{O}[\phi']$ is an arbitrary operator. So, (37) covers both the arbitrary operator Green's functions as well as arbitrary ordinary Green's functions. Specifically, for $\mathcal{O}_1[\phi'] = h'_{ab} h'_{cd}$ (37) describes the gauge graviton propagator, however, for $\mathcal{O}_2[\phi'] = h'_{ab} \bar{c}'^c c'_c$ it describes the 3-point propagator. We want to express the Green's function (G) of perturbative gravity entirely in terms of the linear type gauge Green's functions (and possibly involving vertices from $s\phi$). So we define

$$G(\kappa) = \int \mathcal{D}\phi \mathcal{O}[\phi(\kappa), \kappa] e^{i(S_L[\phi] + S_1[\phi, \kappa])}, \quad (38)$$

where the form of operator $\mathcal{O}[\phi(\kappa), \kappa]$ demands

$$\frac{dG}{d\kappa} = 0. \quad (39)$$

Under FFBRST transformation ($\kappa = 1$), it reflects that

$$G(1) = \int \mathcal{D}\phi' \mathcal{O}[\phi', 1] e^{iS_{NL}[\phi']}, \quad (40)$$

which coincides with (37), where as at $\kappa = 0$ this reads

$$G(0) = \int \mathcal{D}\phi \mathcal{O}[\phi, 0] e^{iS_L[\phi]}, \quad (41)$$

and is numerically equal to (40). Now, we need to determine the form of $\mathcal{O}[\phi(\kappa), \kappa]$ in (38) so that the condition (39) gets satisfied. For this purpose, we perform the field transformation from $\phi(\kappa)$ to $\phi(\kappa + d\kappa)$ through infinitesimal field-dependent BRST transformation defined in (2) which leads

$$\begin{aligned} G(\kappa) &= \int \mathcal{D}\phi(\kappa + d\kappa) \frac{J(\kappa + d\kappa)}{J(\kappa)} \left(\mathcal{O}[\phi(\kappa + d\kappa), \kappa + d\kappa] - s\phi\Theta' \frac{\delta\mathcal{O}}{\delta\phi} d\kappa + \frac{\partial\mathcal{O}}{\partial\kappa} d\kappa \right) \times \\ &\quad \left(1 - i \frac{dS_1}{d\kappa} d\kappa \right) e^{iS_L[\phi(\kappa + d\kappa)] + iS_1[\phi(\kappa + d\kappa), \kappa + d\kappa]}, \\ &= \int \mathcal{D}\phi(\kappa + d\kappa) \left(1 + \frac{1}{J} \frac{dJ}{d\kappa} d\kappa \right) \left(\mathcal{O}[\phi(\kappa + d\kappa), \kappa + d\kappa] - s\phi\Theta' \frac{\delta\mathcal{O}}{\delta\phi} d\kappa + \frac{\partial\mathcal{O}}{\partial\kappa} d\kappa \right) \times \\ &\quad \left(1 - i \frac{dS_1}{d\kappa} d\kappa \right) e^{iS_L[\phi(\kappa + d\kappa)] + iS_1[\phi(\kappa + d\kappa), \kappa + d\kappa]}, \\ &= G(\kappa + d\kappa), \end{aligned} \quad (42)$$

iff

$$\int \mathcal{D}\phi(\kappa) \left(\left[\frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1}{d\kappa} \right] \mathcal{O}[\phi(\kappa), \kappa] - s\phi\Theta' \frac{\delta\mathcal{O}}{\delta\phi} + \frac{\partial\mathcal{O}}{\partial\kappa} \right) e^{iS_L[\phi(\kappa)] + iS_1[\phi(\kappa), \kappa]} = 0. \quad (43)$$

So we get precisely correct expression (43) for replacing Jacobian of path integral measure in Green's function of quantum gravity as e^{iS_1} in place of incorrect one (36).

Exploiting the information of above expression, the required condition for κ -independence of G is

$$\begin{aligned} &\int \mathcal{D}\phi(\kappa) e^{iS_L[\phi(\kappa)] + iS_1[\phi(\kappa), \kappa]} \left(\frac{\partial\mathcal{O}}{\partial\kappa} + \int (\nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}) \Theta' \frac{\delta\mathcal{O}}{\delta h_{ab}} - \int c_b \nabla^b c_a \Theta' \frac{\delta\mathcal{O}}{\delta c_a} \right. \\ &\quad \left. + \int [b_a - \kappa \tilde{c}^b \nabla_b c^a] \Theta' \frac{\delta\mathcal{O}}{\delta \tilde{c}_a} \right) = 0. \end{aligned} \quad (44)$$

Now, if we construct the operator \mathcal{O} to satisfy

$$\begin{aligned} &\frac{\partial\mathcal{O}}{\partial\kappa} + \int (\nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}) \Theta' \frac{\delta\mathcal{O}}{\delta h_{ab}} - \int c_b \nabla^b c_a \Theta' \frac{\delta\mathcal{O}}{\delta c_a} \\ &\quad + \int [b_a - \kappa \tilde{c}^b \nabla_b c^a] \Theta' \frac{\delta\mathcal{O}}{\delta \tilde{c}_a} = 0. \end{aligned} \quad (45)$$

Then condition (44) automatically gets satisfied. Now, we consider a new set of fields $(\tilde{h}_{ab}, \tilde{c}_a, \tilde{\tilde{c}}_a, \tilde{b}_a)$ having following infinitesimal field-dependent BRST transformation:

$$\begin{aligned} \frac{\delta \tilde{h}_{ab}}{\delta \kappa} &= (\nabla_a \tilde{c}_b + \nabla_b \tilde{c}_a + \mathcal{L}_{(\tilde{c})} \tilde{h}_{ab}) \Theta'[\tilde{\phi}], \\ \frac{\delta \tilde{c}^a}{\delta \kappa} &= -\tilde{c}_b \nabla^b \tilde{c}^a \Theta'[\tilde{\phi}], \\ \frac{\delta \tilde{\tilde{c}}^a}{\delta \kappa} &= \tilde{B}^a \Theta'[\tilde{\phi}], \\ \frac{\delta \tilde{B}^a}{\delta \kappa} &= 0, \end{aligned} \quad (46)$$

where $\tilde{B}^a = \tilde{b}^a - \kappa \tilde{c}^b \nabla_b \tilde{c}^a$. These new fields satisfy the following boundary condition: $\phi(\tilde{1}) = \phi(1)$. The condition (45) for $\mathcal{O}[\tilde{\phi}(\kappa), \kappa]$ instead of $\mathcal{O}[\phi(\kappa), \kappa]$ reads

$$\frac{d\mathcal{O}[\tilde{\phi}(\kappa), \kappa]}{d\kappa} = 0. \quad (47)$$

Now utilizing $\mathcal{O}[\tilde{\phi}(1), 1] = \mathcal{O}[\phi(1), 1] = \mathcal{O}[\phi']$ we obtain

$$\mathcal{O}[\tilde{\phi}(\kappa), \kappa] = \mathcal{O}[\phi'], \quad (48)$$

which tells us how the operator $\mathcal{O}[\phi(\kappa), \kappa]$ evolves. To derive FFBRST transformation corresponding to (46), we first define the modification in f of (5) as follows,

$$f[\tilde{\phi}, \kappa] = f_1[\tilde{\phi}] + \kappa f_2[\tilde{\phi}]. \quad (49)$$

Therefore,

$$\frac{d\Theta'[\tilde{\phi}(\kappa)]}{d\kappa} = (f_1[\tilde{\phi}] + \kappa f_2[\tilde{\phi}])\Theta'[\tilde{\phi}(\kappa)] \quad (50)$$

Performing integration from 0 to κ ,

$$\Theta'[\tilde{\phi}(\kappa)] = \Theta[\phi] \exp\left(\kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]\right). \quad (51)$$

Similarly, integrating (46) we get the FFBRST transformation, written compactly as,

$$\begin{aligned} \phi' &= \phi + \left[(\tilde{\delta}_1[\phi] + \tilde{\delta}_2[\phi]) \int d\kappa \exp\left(\kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]\right)\right] \Theta'[\phi], \\ &= \phi + \delta\phi[\phi]. \end{aligned} \quad (52)$$

Now we apply FFBRST transformation (52) on Green's function in non-linear gauge (37)

$$\begin{aligned} G &= \int \mathcal{D}\phi' \mathcal{O}[\phi'] e^{iS_{NL}[\phi']}, \\ &= \int \mathcal{D}\phi \mathcal{O}[\phi + \delta\phi[\phi]] e^{iS_L[\phi]}, \\ &= \int \mathcal{D}\phi \mathcal{O}[\phi] e^{iS_L[\phi]} \\ &\quad + \int \mathcal{D}\phi \left[(\tilde{\delta}_1[\phi] + \tilde{\delta}_2[\phi]) \int d\kappa \exp\left(\kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]\right)\right] \Theta'[\phi] \frac{\delta\mathcal{O}[\phi]}{\delta\phi} e^{iS_L[\phi]}. \end{aligned} \quad (53)$$

Further, it can be written by

$$\langle \mathcal{O} \rangle_{NL} = \langle \mathcal{O} \rangle_L + \int_0^1 d\kappa \int \mathcal{D}\phi (\tilde{\delta}_1[\phi] + \tilde{\delta}_2[\phi]) \Theta'[\phi] \frac{\delta\mathcal{O}[\phi]}{\delta\phi} e^{iS_M}, \quad (54)$$

where $iS_M = iS_L + \kappa f_1[\phi] + \frac{\kappa^2}{2} f_2[\phi]$. In this way, we establish the connection between the Green's function in two gauges in perturbative quantum gravity.

V. CONCLUDING REMARKS

In this work, unlike to the usual FFBRST transformation we have demonstrated the different FFBRST transformation in case of perturbative quantum gravity to relate the arbitrary Green's functions of the

theory corresponding to two different gauges. For concreteness, we have considered the linear and the non-linear gauges from the point of view of their common usage in gravity theory. The Green's functions in non-linear gauge in the theory of perturbative quantum gravity is expressed as a series in terms of those in linear gauges. In this context we have shown the remarkable difference between the the modified FFBRST transformation and the usual one. Further, being related to the usual FFBRST formulation, this modified FFBRST transformation is obtained by integration of (46). We hope that the final result putted in a simple form will be very useful from computational point of view in the theory of perturbative quantum gravity.

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